Velocity Field Control with Energy Compensation toward Therapeutic Exercise

Takaaki Shogaki, Takahiro Wada and Yoshio Fukui

Abstract—Human-machine systems such as rehabilitation machines are required to be safe for human bodies as well as achieving a given operating task. Passivity-based controller such as Passive Velocity Field Control has an advantage to realize safe operation of human machine system. On the other hand, behaving actively toward external environment including human bodies is required to realize a given task. However, behaving actively is difficult for such passivity-based controller. The problem of the present paper is to ensure that a manipulator behaves passively toward external force when kinetic energy is greater than or equal to given threshold and actively otherwise. The present paper proposed a velocity field control method with energy compensation mechanism. Numerical simulations demonstrated that the closed loop system behaved passively toward external force basically, and the proposed method inhibited decrease of the kinetic energy of the closed loop system by the dissipative external force.

I. INTRODUCTION

Human-machine systems in which the human has physical interaction with the machine, such as a rehabilitation machine, has drawn much attention. Such systems are required to ensure safety of human bodies and to achieve a given operating task simultaneously.

Ensuring passivity of the closed loop system is an effective way to increase safety. For example, in the context of a power assist system, virtual power limiter method has been proposed, in which the additional conservative controller is smoothly connected to existing another controller according to the virtual power monitors’ output to guarantee passivity [1]. In the field of rehabilitation robotics such as support and control input to the virtual power monitors’ output to guarantee passivity [1]. In the field of rehabilitation robotics such as support and control input to the virtual power monitors’ output to guarantee passivity [1]. In the field of rehabilitation robotics such as support and control input to the virtual power monitors’ output to guarantee passivity [1]. In the field of rehabilitation robotics such as support and control input to the virtual power monitors’ output to guarantee passivity [1].

II. PROBLEM STATEMENT

A. n Link Manipulator

The dynamic equation of n link manipulator is modeled as

\[ \dot{\mathbf{M}}(\mathbf{q}) \ddot{\mathbf{q}} + \frac{1}{2} \ddot{\mathbf{M}}(\mathbf{q}) \dot{\mathbf{q}} + \mathbf{S}(\mathbf{q}, \dot{\mathbf{q}}) \dot{\mathbf{q}} = \mathbf{\tau} + \mathbf{\tau}_e. \]  

(1)

where \( \mathbf{q} = [q_1, q_2, \ldots, q_n]^T \) denotes joint displacement, \( \mathbf{\tau} \) and \( \mathbf{\tau}_e \) denote generalized force representing control input and external generalized force, respectively, positive definite symmetric \( \mathbf{M} \) is the inertia matrix, and time derivative of the inertia matrix \( \dot{\mathbf{M}} \) and skew-symmetric matrix \( \mathbf{S} \) are given by

\[ \dot{\mathbf{M}}(\mathbf{q}(t)) := \frac{d}{dt} \mathbf{M}(\mathbf{q}(t)), \]

\[ \mathbf{S}(\mathbf{q}, \dot{\mathbf{q}}) := \frac{1}{2} \dot{\mathbf{M}}(\mathbf{q}) \dot{\mathbf{q}} - \frac{1}{2} \dot{\mathbf{q}}^T \dot{\mathbf{M}}(\mathbf{q}) \dot{\mathbf{q}}. \]  

(3)

Without loss of generality, we suppose the robot has n rotational joints in the present paper, thus, the joint displacement and the generalized force are joint angle and joint torque, respectively.

B. Augmented Mechanical System

Let us introduce a fictitious flywheel whose dynamics is given by

\[ m_f \ddot{\theta}_f(t) = \tau_f(t), \]  

(4)

It was also applied to rehabilitation exercise machine for wrist motion [10].

On the other hand, there exists a robotic therapy in which the robot moves human body actively while the human patient does not move actively. Continuous Passive Motion (CPM) in therapeutic exercise [11] is one of the examples. In such cases, a control system should be active to move human bodies with a certain amount of inertia with dissipative element in their joint. It is impossible for fully passive controls to achieve such control.

The purpose of the present paper is to propose a velocity field control method with energy compensation mechanism by extending PVFC [6]. The closed loop system with the method behaves actively toward external force when kinetic energy of the closed loop system is lower than a given threshold and behaves passively otherwise. The effectiveness of the proposed method is confirmed by numerical simulations in two external force conditions: viscous friction force, which imitates the joint resistance of the human patient and a force by a PD feedback, which imitates an unexpected action of the human.
where $q_f(t)$ is the angle of the flywheel, $\tau_f(t)$ is control torque exerted on the flywheel, and a constant $m_f$ is mass of the flywheel. Dynamics of the augmented system, which consists of the original robot manipulator and the fictitious flywheel, can be represented by

$$M^a(q)\ddot{q}^a + \frac{1}{2}M^a(q, \dot{q}^a)\dot{q}^a + S^a(q, \dot{q}^a)\dot{q}^a = \tau^a + \tau_e^a, \quad (5)$$

$$M^a(q) := \begin{bmatrix} M(q) & 0_{nx1} \\ 0_{1nx} & m_f \end{bmatrix}, \quad (6)$$

$$S^a(q, \dot{q}^a)\dot{q}^a := \frac{1}{2}M^a(q)\dot{q}^a - \frac{1}{2}\frac{\partial}{\partial q^a}(\dot{q}^a M^a(q)\dot{q}^a), \quad (7)$$

where joint angle vector $q^a$, control input $\tau^a$, and external torque exerted on the augmented system $\tau_e^a$ are defined as follows:

$$q^a := \begin{bmatrix} q \\ q_f \end{bmatrix}, \quad \tau^a := \begin{bmatrix} \tau \\ \tau_f \end{bmatrix}, \quad \tau_e^a := \begin{bmatrix} \tau_e \\ 0 \end{bmatrix}. \quad (8)$$

C. Desired Velocity Field

The desired velocity field that should be followed by the augmented system (5) is defined by $V^a(q) := [V_T(q), V_f(q)]^T$, where $V(q)$ is the desired velocity field that is designed in advance according to the given task. Desired velocity of flywheel $V_f$ is defined by

$$V_f(q) := \frac{2}{m_f} \left( E^a - \frac{1}{2}V(q)^T M(q) V(q) \right), \quad (9)$$

where $E^a$ is a sufficiently large positive constant so that the inside of the square root is positive. Kinetic energy of the augmented system $k^a(q, \dot{q}^a)$ is defined as

$$k^a(q, \dot{q}^a) := k(q, \dot{q}^a) + k_f(q_f), \quad (10)$$

where kinetic energy of the manipulator $k(q, \dot{q}^a)$ and the flywheel $k_f(q_f)$ are expressed as follows:

$$k^a(q, \dot{q}^a) = \frac{1}{2}\dot{q}^a M^a(q)\dot{q}^a, \quad (11)$$

$$k_f(q_f) := \frac{1}{2}m_f \dot{q}_f^2. \quad (12)$$

Note that $k^a(q, V^a(q)) = E^a$ holds by (9) when the manipulator exactly follows the desired velocity field.

D. Problem

The problem in the present paper is to design a velocity field tracking control for human-machine system that maintains kinetic energy of the closed loop system even with the dissipative external force such as the joint resistance of human patient. It also required to behave passively toward the external force that increases kinetic energy largely for safety of human. In particular, our problem is to design a control torque $\tau^e(q, \dot{q}^a)$ that keeps energy level when it is smaller than a predetermined value and behaves passively toward the external force otherwise.

III. PROPOSED METHOD

A. Control Law

We propose a control method to follow given velocity field based on PFVC [6]:

$$\tau^{a}(q, \dot{q}^{a}) := G^{a}(q, \dot{q}^{a})\dot{q}^{a} + R^{a}(q, \dot{q}^{a})\dot{q}^{a} + \tau^{e}(q, \dot{q}^{a}), \quad (13)$$

where skew-symmetric matrices $G^a$ and $R^a$ are defined as follows:

$$G^a(q, \dot{q}^a) := \frac{1}{2E^a}(w^a(q, \dot{q}^a)Q^a(q)^T - Q^a(q)w^a(q, \dot{q}^a)^T), \quad (14)$$

$$R^a(q, \dot{q}^a) := \gamma(Q^a(q)P^a(q, \dot{q}^a)^T - P^a(q, \dot{q}^a)Q^a(q)^T), \quad (15)$$

where $\gamma(>0)$ is a control gain, $P^a$ is current momentum of the augmented system defined as (16), $Q^a$ is momentum of the augmented system (5) assuming that it exactly follows the desired velocity defined as (17), $w^a$ is inverse dynamics of the augmented system assuming that it exactly follows the desired velocity that is defined as (18).

$$P^a(q, \dot{q}^a) := M^a(q, \dot{q}^a) \quad (16)$$

$$Q^a(q) := M^a(q)V^a(q) \quad (17)$$

$$w^a(q, \dot{q}^a) := M^a(q)V^a(q) + \frac{1}{2}M^a(q)V^a(q) + \frac{1}{2}S^a(q, \dot{q}^a)V^a(q) \quad (18)$$

$\tau^{e}(q, \dot{q}^a)$ in (13) denotes the energy control law by the input to the fictitious flywheel defined as

$$\tau^{e}(q, \dot{q}^a) := \begin{bmatrix} 0_{nx1} \\ \eta \, s \left( k_d^a - k^a(q, \dot{q}^a) \right) \dot{q}_f \end{bmatrix}, \quad (19)$$

where, $\eta(>0)$ is a constant gain, $k_d^a$ is the desired value of the energy of the augmented system and $s(\cdot)$ is a class $C^1$ function defined as:

$$s(x) := \begin{cases} 0.0 & (x \leq 0) \\ \frac{1}{2} \left( 1 - \cos \left( \frac{\pi}{2} x \right) \right) & (0 < x \leq a) \\ 1.0 & (x > a) \end{cases} \quad (20)$$

where $a(>0)$ is a constant. Equation (20) compensates decrease of the kinetic energy in the case of $k_d^a - k^a(q(t), \dot{q}(t)) > 0$, therefore this is referred to as energy compensation mechanism.

B. Discussion

The proposed control is equivalent to the PVFC [6] when $\tau^{e}(q(t), \dot{q}^a(t))$ is set to be 0 for all $t$.

Masuyama et al., [12] proposed a similar energy control method. As the major difference, our proposed method (13) through (20) can switch the behavior of the augmented system (5) depending on the external force, but passivity of the method in [12] is not guaranteed.

In the proposed method (13) through (20), $\tau^{e}$ becomes 0 when $k_d^a - k^a(q(t), \dot{q}(t)) \leq 0$. Thus, the closed loop system
behaves passively depending on external force that is same as the conventional method [6]. On the other hand, in the case that \( k_d^a - k^a(q(t), \dot{q}(t)) > 0 \) is satisfied, virtual torque is inputted to the fictitious flywheel, thus, it is expected that \( k^a(q(t), \dot{q}(t)) \) converges to \( k_d^a \).

IV. SIMULATION

In this section, we confirm the effectiveness of the proposed method via numerical simulations. In the simulation, we consider a 2 link manipulator shown in Fig. 1. The task is that the end-effector of the manipulator traces a circle.

A. Desired Velocity Field

Desired velocity field for the task a circle is given as follows:

\[
V_P := \begin{bmatrix}
\sin \theta & (r_d - r) \cos \theta \\
-\cos \theta & (r_d - r) \sin \theta
\end{bmatrix}
\begin{bmatrix}
G_T \\
G_R
\end{bmatrix},
\]

where \( r(z(q)) := \sqrt{(x_0 - x(t))^2 + (y_0 - y(t))^2} \),

\[
(21)
\]

\[
r(z(q)) := \sqrt{(x_0 - x(t))^2 + (y_0 - y(t))^2},
\]

(22)

where \( z(q) = [x(q), y(q)]^T \) denotes the end-effector’s position described by Cartesian coordinate \( o-XY \) (Fig. 1), \( \theta \) denotes the central angle of the circle measured from the \( X \)-axis and the end-effector’s current position, \( r_d \) denotes the radius of the desired circle, \( r \) denotes the length from the central position of the desired circle to end-effector’s current position, \( (x_0, y_0) \) is the position of the center of the desired circle, and \( G_T \) and \( G_R \) are scalars that determine the direction of the velocity vector field.

The desired velocity vector \( V_P \) described in the task coordinate system \( o-XY \) is converted into the desired angular velocity field \( V(q) \) which is described by joint angle coordinate system by

\[
V(q(t)) = J(q(t))^{-1}V_P(z(t)),
\]

(23)

where \( J(q) := \partial z/\partial q \) denotes Jacobian matrix of \( z \).

B. Conditions

1) Control Condition: We confirm the behavior of the closed loop system with the proposed method and the conventional method (PVFC) toward external force. The proposed method and the conventional method (PVFC) are compared, there are two conditions in \( \tau^e \) as follows:

(a) Proposed method

\[
\tau^e(q, \dot{q}^a) := \eta s(k_d^a - k^a(q, \dot{q}^a)) \dot{q}_f
\]

(24)

(b) Conventional method (PVFC)

\[
\tau^e(q, \dot{q}^a) := 0_{2 \times 1}
\]

(25)

2) External Force Condition: The following two external force conditions are considered.

(a) External force 1: Viscous friction force

In this condition, we consider the following external torque:

\[
\tau_e(t) = \begin{cases}
0 & (0 \leq t < 20) \\
- D \dot{q}(t) & (20 \leq t < 80) \\
0 & (80 \leq t \leq 100).
\end{cases}
\]

(26)

This viscous friction force was introduced to imitate joint resistance of human or spasticity, which supposed to be occurred in a passive motion of the human joints.

(b) External force 2: Force given by PD feedback

In this condition, we consider the following external torque:

\[
\tau_e(t) = \begin{cases}
0 & (0 \leq t < 20) \\
J(q(t))^T(K_P(z_d - z(t)) - K_D \ddot{z}(t)) & (20 \leq t \leq 60),
\end{cases}
\]

(27)

where \( z_d \) is the virtual desired position of the PD feedback force, \( (K_P, K_D) \) is a control gain. In this simulation, we set \( (K_P, K_D) = (500.0, 50.0) \), \( z_d = [0.2, -0.2]^T \). This PD feedback control force is introduced to imitate the unexpected reaction of human.

(c) External force 3: Force given by PD feedback for an instant

In this condition, we consider the following external torque:

\[
\tau_e(t) = \begin{cases}
0 & (0.0 \leq t < 20.0) \\
J(q(t))^T(K_P(z_d - z(t)) - K_D \ddot{z}(t)) & (20.0 \leq t < 20.1) \\
0 & (20.1 \leq t \leq 60.0),
\end{cases}
\]

(28)

where \( z_d \) is the virtual desired position of the PD feedback force, \( (K_P, K_D) \) is a control gain. In this simulation, we set \( (K_P, K_D) = (500.0, 50.0, 0.0) \), \( z_d = [0.2, -0.2]^T \). This PD feedback control force is introduced to imitate the unexpected reaction of human for an instant.

3) Parameter Setting: The desired value of the energy of the augmented system is set as \( k_d^a = k^a(q(0), \dot{q}^a(0)) = 0.78 \). The gains are set as \( \eta = 0.5 \) and \( a = 0.01 \).

Let the link length \( (\text{Link 1, Link 2}) \) be \([0.4, 0.4][m] \) and the link mass be \([2.8, 2.8][k] \). The moment of inertia of the links around the center of gravity is set as \([0.2, 0.2][k] \) and center of gravity to be center of the link.
Initial values of the joint angle and the angular velocity are set as \((q_1(0), q_2(0), \dot{q}_1(0), \dot{q}_2(0)) = (-45, 90, 0)[\text{deg}]\) and \((\dot{q}_1(0), \dot{q}_2(0), \ddot{q}_1(0)) = (-70, 50, 50)[\text{deg/s}]\), respectively. The mass of the flywheel is set as \(m = 300\text{[kg]}\) and the control gain are \(\gamma = 0.1\). The kinetic energy of the augmented system is equal to a predetermined value, and actively toward external force when the kinetic energy of the closed loop system is greater than or equal to a predetermined value, and actively toward external force otherwise.

The numerical simulation results demonstrated that decrease of the end-effector’s velocity was inhibited with the proposed method when viscous friction force was exerted. It was also shown that the closed loop system behaves passively toward external force when high energy level situation \((k_a(q(t), \dot{q}(t)) > 0)\) same as PVFC [6]. These results suggest that when we apply the method to the therapeutic exercise such as the CPM, the robot behaves passive to the large force input such as patients’ unintended or escaping motion for avoiding pain etc, and also behaves in accordance with therapeutic motion for patient even under viscous friction or other resistance force that dissipates energy of the robot.

The proposed method will be implemented to the real robotic system to examine its validity. With the proposed method, the end-effector trajectory is close to the desired circle. It demonstrates that the end-effector of the manipulator moved along the desired velocity field.

V. CONCLUSION

In this paper, a velocity field control with energy compensation mechanism was proposed. The energy compensation mechanism works passively toward external force when the kinetic energy of the closed loop system is greater than or equal to a predetermined value, and actively toward external force otherwise.

The numerical simulation results demonstrated that decrease of the end-effector’s velocity was inhibited with the proposed method when viscous friction force was exerted. It was also shown that the closed loop system behaves passively toward external force when high energy level situation \((k_a(q(t), \dot{q}(t)) > 0)\) same as PVFC [6]. These results suggest that when we apply the method to the therapeutic exercise such as the CPM, the robot behaves passive to the large force input such as patients’ unintended or escaping motion for avoiding pain etc, and also behaves in accordance with therapeutic motion for patient even under viscous friction or other resistance force that dissipates energy of the robot.

The proposed method will be implemented to the real robotic system to examine its validity. With the proposed method, the end-effector’s velocity is increased after removing external force. Unintended acceleration of the robot may
occur. As an important future study, this phenomenon should be avoided for human patients’ safety.

REFERENCES


Fig. 8. Kinetic energy (External force 2, Proposed method)

Fig. 9. Kinetic energy (External force 2, Conventional method)

Fig. 10. Comparison of end effector velocity of the manipulator (External force 2)

Fig. 11. Hand point track (External force 2, Proposed method)

Fig. 12. Hand point track (External force 2, Conventional method)

Fig. 13. Kinetic energy (External force 3, Proposed method)

Fig. 14. Kinetic energy (External force 3, Conventional method)

Fig. 15. Comparison of end effector velocity of the manipulator (External force 3)

Fig. 16. Hand point track (External force 3, Proposed method)

Fig. 17. Hand point track (External force 3, Conventional method)